2501/103 2508/103
2502/103 2509/103
2503/103
ENGINEERING MATHEMATICS I
Oct./Nov. 2019
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN MECHANICAL ENGINEERING (PLANT OPTION) (PRODUCTION OPTION) DIPLOMA IN AUTOMOTIVE ENGINEERING DIPLOMA IN WELDING AND FABRICATION DIPLOMA IN CONSTRUCTION PLANT ENGINEERING

MODULE I

ENGINEERING MATHEMATICS I

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/scientific calculator.
*Answer **FIVE** of the following **EIGHT** questions.*
All questions carry equal marks.
Maximum marks for each part of a question are indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Given that $\sin A = \frac{5}{13}$ and $\cos B = \frac{6}{10}$, where A and B are acute angles, determine without using Mathematical tables or calculator, the value of:

$(\cos 90 \cos A + \sin 90 \sin A)(\sin 90 \cos B - \cos 90 \sin B)$

(i) $\cos(90 - A)\sin(90 - B)$;

(ii) $\sec^2 A \operatorname{cosec}^2 B$. $\frac{1}{\cos^2 A} \times \frac{1}{\sin^2 B} = \frac{1}{\left(\frac{12}{13}\right)^2} \times \frac{1}{\left(\frac{6}{10}\right)^2}$ $\quad 1 \times \left(\frac{13}{12}\right)^2 \times 1 \times \left(\frac{10}{6}\right)^2$ (7 marks)

$\frac{169}{144} \times \frac{100}{36} = \frac{16900}{5{,}184}$

(b) Prove the identities:

(i) $2\cos^2\frac{\theta}{2}\sec\theta = \sec\theta + 1$;

(ii) $\frac{\cos\theta}{\sqrt{1+\tan^2\theta}} + \frac{\sin\theta}{\sqrt{1+\cot^2\theta}} = 1.$

(6 marks)

(c) Solve the equation $\cos 2\theta + \cos\theta + 1 = 0$ for values for θ between 0° and 360° inclusive. (7 marks)

2. (a) Given the complex numbers $z_1 = 2 + j4, z_2 = 3 - j4$ *and* $z_3 = -1 + j2$, determine:

$2 + j4 + 3 - j4 + 1 - j2$

(i) $z_1 + z_2 - z_3$;

$6 - j2$

(ii) $z_2 + \frac{z_1 z_3}{z_1 + z_3}$

(6 marks)

(b) Solve the equation $z^4 - 3 - j4 = 0$, giving the answers in exponential form. (9 marks)

(c) Three coplanar forces acting at a point are given by $8 < 45^\circ N, 7 < 120^\circ N$ and $14 < 210^\circ N$. Determine using complex numbers the magnitude and direction of the resultant force. (5 marks)

3. (a) Simplify:

$(x^2 - 1)^2 \times \sqrt{(x+1)} \div (x-1)^{\frac{-3}{2}}$. (3 marks)

$\log (x-3)(x+3) = \log (x+3)^2$

$(x-3)(x+3) = (x+3)(x+3)$

$x^2 + 3x - 3x = x^2 + 6x + 9$

$x^2 - 6x = 9$

(b) Solve the equations:

(i) $\log(x-3) + \log(x+3) = 2\log(x+3)$;

(ii) $4(16^{x+4}) \times 5.2^{2x} = 13$.

(9 marks)

(c) Solve the equation $\log_5 y + \log_y 25 = 3$. (8 marks)

$\log_5 y + \log_y 25 = 3$

$\frac{\log y}{\log 5} + \frac{\log 25}{\log y} = 3$

$\frac{\log_5 y}{\log_5 5} + \frac{\log_5 25}{\log_5 y} = 3$

$\log_5 y + \frac{2}{\log_5 y} = 3$

$\log_5 y \to n$

$n + \frac{2}{n} = 3$ so $n = 1$

$\log_5 y = n = 5^n = y$

$5^1 = y$ or

$y = 5$

1. (a) Given that $\sin A = \frac{5}{13}$ and $\cos B = \frac{6}{10}$, where A and B are acute angles, determine without using Mathematical tables or calculator, the value of:

(i) $\cos(90 - A)\sin(90 - B)$;

(ii) $\sec^2 A \ \operatorname{cosec}^2 B$.

(7 marks)

(b) Prove the identities:

(i) $2\cos^2\frac{\theta}{2}\sec\theta = \sec\theta + 1$;

(ii) $\dfrac{\cos\theta}{\sqrt{1+\tan^2\theta}} + \dfrac{\sin\theta}{\sqrt{1+\cot^2\theta}} = 1$.

(6 marks)

(c) Solve the equation $\cos 2\theta + \cos\theta + 1 = 0$ for values for θ between 0° and 360° inclusive. (7 marks)

2. (a) Given the complex numbers $z_1 = 2 + j4, z_2 = 3 - j4$ *and* $z_3 = -1 + j2$, determine:

(i) $z_1 + z_2 - z_3$;

(ii) $z_2 + \dfrac{z_1 z_3}{z_1 + z_3}$.

(6 marks)

(b) Solve the equation $z^4 - 3 - j4 = 0$, giving the answers in exponential form. (9 marks)

(c) Three coplanar forces acting at a point are given by $8 < 45^\circ N, 7 < 120^\circ N$ and $14 < 210^\circ N$. Determine using complex numbers the magnitude and direction of the resultant force. (5 marks)

3. (a) Simplify:

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(i) $\log(x-3) + \log(x+3) = 2\log(x+3)$;

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(9 marks)

(c) Solve the equation $\log_5 y + \log_y 25 = 3$. (8 marks)

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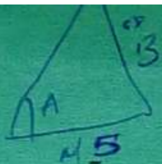

1\. (a) Given that $\sin A = \frac{5}{13}$ and $\cos B = \frac{6}{10}$, where A and B are acute angles, determine without using Mathematical tables or calculator, the value of:

(i) $\cos(90 - A)\sin(90 - B)$;

(ii) $\sec^2 A \ \text{cosec}^2 B$.

(7 marks)

(b) Prove the identities:

(i) $2\cos^2\frac{\theta}{2}\sec\theta = \sec\theta + 1$;

(ii) $\frac{\cos\theta}{\sqrt{1+\tan^2\theta}} + \frac{\sin\theta}{\sqrt{1+\cot^2\theta}} = 1$.

(6 marks)

(c) Solve the equation $\cos 2\theta + \cos\theta + 1 = 0$ for values for θ between 0° and 360° inclusive. (7 marks)

2\. (a) Given the complex numbers $z_1 = 2 + j4$, $z_2 = 3 - j4$ *and* $z_3 = -1 + j2$, determine:

(i) $z_1 + z_2 - z_3$;

$(2+j4)+(3-j4)$

(ii) $z_2 + \frac{z_1 z_3}{z_1 + z_3}$.

$5+j0-(-1+j2)$ $\underline{6+2j}$

(6 marks)

(b) Solve the equation $z^4 - 3 - j4 = 0$, giving the answers in exponential form. (9 marks)

(c) Three coplanar forces acting at a point are given by $8 < 45^\circ N, 7 < 120^\circ N$ and $14 < 210^\circ N$. Determine using complex numbers the magnitude and direction of the resultant force. (5 marks)

3\. (a) Simplify:

$(x^2-1)^2 \times \sqrt{(x+1)} \div (x-1)^{\frac{-3}{2}}$. (3 marks)

(b) Solve the equations:

(i) $\log(x-3) + \log(x+3) = 2\log(x+3)$;

(ii) $4(16^{x+4}) \times 5.2^{2x} = 13$.

(9 marks)

(c) Solve the equation $\log_5 y + \log_y 25 = 3$. (8 marks)

4. (a) Use the binomial theorem to expand $(1+x-x^2)^8$ up to the term in x^3. (4 marks)

(b) (i) Show that if x^3 and higher powers may be neglected

$$\frac{\sqrt{1+2x}}{1-x} = 1 + 2x + \frac{3}{2}x^2.$$

(ii) Hence, evaluate $\frac{\sqrt{1.06}}{0.97}$, correct to four decimal places. (10 marks)

(c) Pressure P and volume V of a gas are related by the equation $C = PV^3$, where C is a constant. Determine, using the binomial theorem, the approximate change in C when P increases by 2% and V decreases by 1.1%. (6 marks)

5. (a) The second, fourth and eighth terms of an arithmetic progression are in geometric progression and the sum of the third and fifth terms is 20. Determine the:

(i) first term;

(ii) common difference.

(7 marks)

(b) Solve the equation $\frac{1}{y^2} - \frac{5}{y} = -6$. (5 marks)

(c) In a system of forces, the relationship between three forces F_1, F_2 and F_3 in newtons is given by the simultaneous equations:

$$\begin{aligned} 2F1 + 3F2 - 4F3 &= 26 \\ F1 - 5F2 - 3F3 &= -87 \\ -7F1 + 2F2 + 6F3 &= 12 \end{aligned}$$

Use elimination method to solve the equations. (8 marks)

6. (a) Simplify:

(i) $\frac{21!}{7!14!} + \frac{21!}{8!13!}$;

(ii) $\frac{(n+1)!}{n-2!}$.

(5 marks)

(b) A box contains 5 red, 4 white and 3 blue balls. Three balls are drawn at random. Determine the number of ways of selecting one ball of different colour.

(5 marks)

4. (a) Use the binomial theorem to expand $(1+x-x^2)^8$ up to the term in x^3. (4 marks)

(b) (i) Show that if x^3 and higher powers may be neglected

$$\frac{\sqrt{1+2x}}{1-x} = 1+2x+\frac{3}{2}x^2.$$

(ii) Hence, evaluate $\frac{\sqrt{1.06}}{0.97}$, correct to four decimal places. (10 marks)

(c) Pressure P and volume V of a gas are related by the equation $C = PV^3$, where C is a constant. Determine, using the binomial theorem, the approximate change in C when P increases by 2% and V decreases by 1.1%. (6 marks)

5. (a) The second, fourth and eighth terms of an arithmetic progression are in geometric progression and the sum of the third and fifth terms is 20. Determine the:

(i) first term;

(ii) common difference.

(7 marks)

(b) Solve the equation $\frac{1}{y^2}-\frac{5}{y}=-6$. (5 marks)

(c) In a system of forces, the relationship between three forces F_1, F_2 and F_3 in newtons is given by the simultaneous equations:

$$\begin{aligned} 2F1 + 3F2 - 4F3 &= 26 \\ F1 - 5F2 - 3F3 &= -87 \\ -7F1 + 2F2 + 6F3 &= 12 \end{aligned}$$

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6. (a) Simplify:

(i) $\frac{21!}{7!14!}+\frac{21!}{8!13!}$;

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Turn over

2501/103 2508/103
2502/103 2509/103
2503/103
Oct./Nov. 2019

(c) Convert:

(i) $\frac{2}{r} = 1 + 3\cos\theta$ to cartesian form;

(ii) $xy = \frac{1}{2}$ to polar form.

(6 marks)

(d) Two points have coordinates A(2, 8) and B(8, 6). Find the equation of the tangent to the line AB at the point A(2, 8).

(4 marks)

7. (a) Given $f(x) = \frac{-6}{x-8}$, determine $f^{-1}(x)$.

(3 marks)

(b) Prove the identities:

(i) $\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$;

(ii) $\cosh 3x = 4\cosh^3 x - 3\cosh x$;

(iii) $\sinh(x+h) = \sinh x \cosh y + \cosh x \sinh y$.

(11 marks)

(c) Solve the equation $7\sinh x - 5\cosh x = -1$, correct to three decimal places.

(6 marks)

8. (a) Simplify $\frac{1}{3}$ of$\left(5\frac{1}{2} - 1\frac{1}{4}\right) + 1\frac{1}{5} \div \frac{3}{5} - \frac{1}{2}$.

(5 marks)

(b) The first of an arithmetic progression is -12 and the last term is 40. If the sum of the progression is 196, determine the:

(i) number of terms;

(ii) common difference.

(7 marks)

(c) A factory produces 1200 bolts in the first week. If the rise in weekly production is 6%, determine the:

(i) number of weeks required to produce more than 8000 bolts;

(ii) number of bolts produced in the last week.

(8 marks)

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(c) Convert:

(i) $\frac{2}{r} = 1 + 3\cos\theta$ to cartesian form;

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(b) The first of an arithmetic progression is -12 and the last term is 40. If the sum of the progression is 196, determine the:

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(c) A factory produces 1200 bolts in the first week. If the rise in weekly production is 6%, determine the:

(i) number of weeks required to produce more than 8000 bolts;

(ii) number of bolts produced in the last week.

(8 marks)

1200

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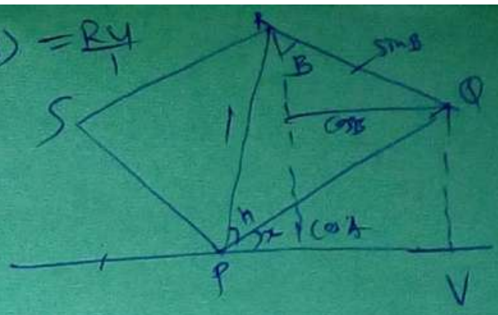
$\sin(A+B) = \frac{RY}{1}$

$PQ = \cos A =$

$PQ = \sin A$

$QPU = B$



(c) Convert:

(i) $\frac{2}{r} = 1 + 3\cos\theta$ to cartesian form;

(ii) $xy = \frac{1}{2}$ to polar form.

(6 marks)

(d) Two points have coordinates A(2, 8) and B(8, 6). Find the equation of the tangent to the line AB at the point A(2, 8). (4 marks)

7. (a) Given $f(x) = \frac{-6}{x-8}$, determine $f^{-1}(x)$. (3 marks)

(b) Prove the identities:

$\tanh 2x = \frac{\sinh 2x}{\cosh 2x} = \frac{2\cosh x \sinh x}{}$

(i) $\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$;

(ii) $\cosh 3x = 4\cosh^3 x - 3\cosh x$;

(iii) $\sinh(x+h) = \sinh x \cosh y + \cosh x \sinh y$.

(11 marks)

$\tanh 2x = \frac{\sinh 2x}{\cosh 2x}$

$= \frac{2\sinh x \cosh x}{\cosh^2 x + \sinh^2 x}$

$= \frac{2\sinh x \cosh x}{\cosh^2 x}$

$= \frac{\cosh^2 x}{\cosh^2 x} + \frac{\sinh^2 x}{\cosh^2 x}$

$= \frac{2\sinh x}{\cosh x}$

$1 + \tanh^2 x$

$= \frac{2\tanh x}{1+\tanh^2 x}$

$\cosh 3x = \cosh(2x + x)$

$\cosh 2x \cosh x + \sinh 2x \sinh x$

$\cosh x(2\cosh^2 x - 1) - \sinh x(2\sinh x \cosh x)$

$2\cosh^3 x - \cosh x - 2\sinh^2 x \cosh x$

$2\cosh^3 x - \cosh x - 2\cosh x(\cosh^2 x - 1)$

$2\cosh^3 x - \cosh x + 2\cosh^3 x + 2\cosh x$

$4\cosh^3 x - 3\cosh x$

(c) Solve the equation $7\sinh x - 5\cosh x = -1$, correct to three decimal places.

(6 marks)

(a) Simplify $\frac{1}{3}$ of$\left(5\frac{1}{2} - 1\frac{1}{4}\right) + 1\frac{1}{5} \div \frac{3}{5} - \frac{1}{2}$. (5 marks)

(b) The first of an arithmetic progression is -12 and the last term is 40. If the sum of the progression is 196, determine the:

(i) number of terms;

(ii) common difference.

(7 marks)

$1200 = 100$
$\times\ 94\ ?$

$\frac{1200 \times 6}{100}$
$= 72$

$1200 = 100\ \%$
$\times\ 6\ \%$

If $1200 = 100\ \%$
$\times\ 106\ \%$

(c) A factory produces 1200 bolts in the first week. If the rise in weekly production is 6%, determine the:

$\frac{1272}{1250}$

(i) number of weeks required to produce more than 8000 bolts;

(ii) number of bolts produced in the last week.

(8 marks)

$1272 = 1$ week
$8000\ X$

$\frac{6}{100} \times 1200$

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$1200 = 100\ \%$
100